1

Who takes Mimy, the cat?
The decisional hardships of a brother and a sister!

I want to take Mimy!

I want to keep Mimy!

Sukhi & Jinder Atwal B.C. brother-sister team off to The Amazing Race. And they are competitive!

They share Mimy, the family cat!

But Jinder is now leaving their hometown and wants to take Mimy with him.

Disclaimer: the story and characters here are made up for pedagogical entertainment ☺

2

Who takes Mimy, the cat?
The decisional hardships of a brother and a sister!

Let' toss this coin 30 times to decide who takes Mimy.

Who gets more than 15 tails takes Mimy.

Disclaimer: the story and characters here are made up for pedagogical entertainment ☺

3

Who takes Mimy, the cat?
The decisional hardships of a brother and a sister!

Your coin? You always cheated in games when we were young! Let's use my coin

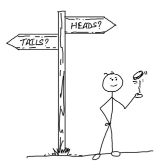
But should I trust you?

Disclaimer: the story and characters here are made up for pedagogical entertainment ☺

4

Sukhi, who took BIOL322, proposes that they use statistics to judge each other coins.

All right, I propose a statistical experiment to test our coins.



Disclaimer: the story and characters here are made up for pedagogical entertainment ☺

5

Sukhi proposes that Jinder takes her coin & she takes his.

Each of us toss each other coins many sets of 30 times and graph the results!

6

Sukhi proposes that Jinder takes her coin & she takes his.

You really don't trust me!

Each of us toss each other coins many sets of 30 times and graph the results!

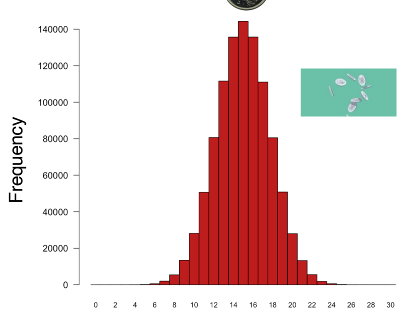
7

Sukhi proposes that Jinder takes her coin & she takes his.

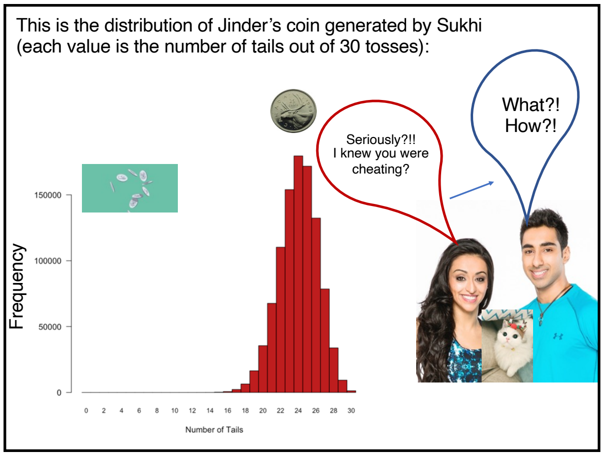
Nope,
I don't!!!

8

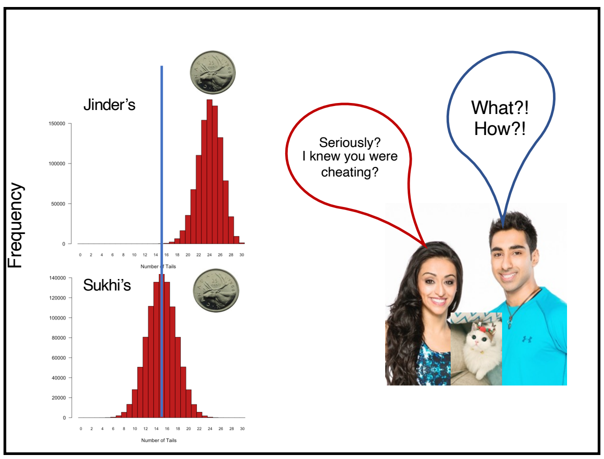
This is the distribution of Sukhi's coin generated by Jinder (each value is the number of tails out of 30 tosses):



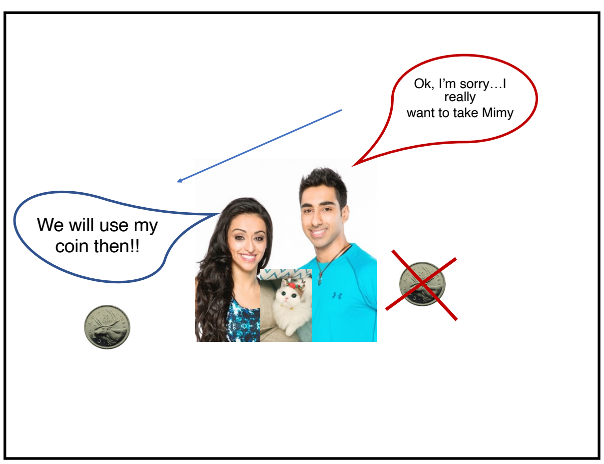
9


This is the distribution of Jinder's coin generated by Sukhi
(each value is the number of tails out of 30 tosses):
Seriously?!!
I knew you were
cheating?
What?!
How?!
Frequency
Number of Tails

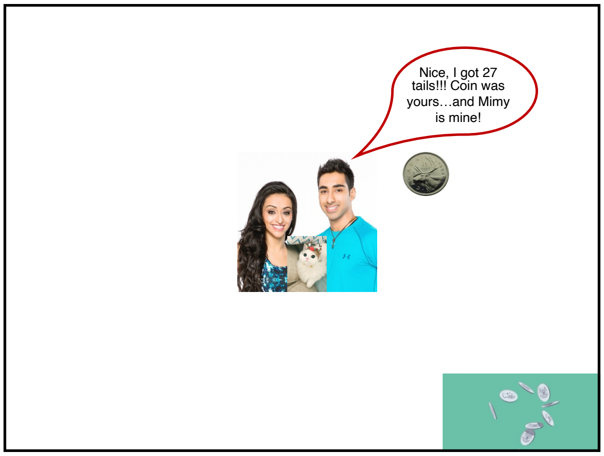
10


Jinder's
Sukhi's
Frequency
Seriously?
I knew you were
cheating?
What?!
How?!

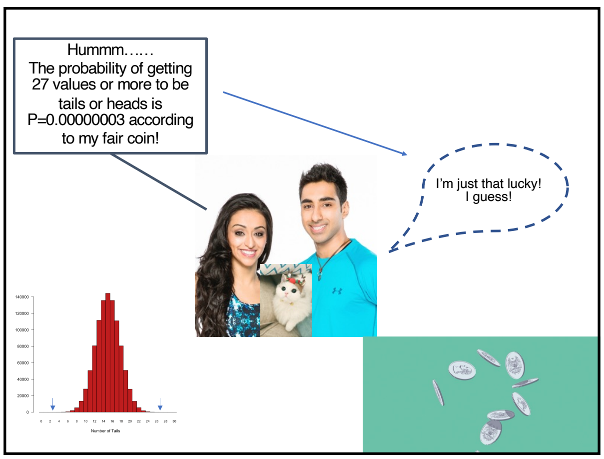
11


Ok, I'm sorry…I
really
want to take Mimy
We will use my
coin then!!

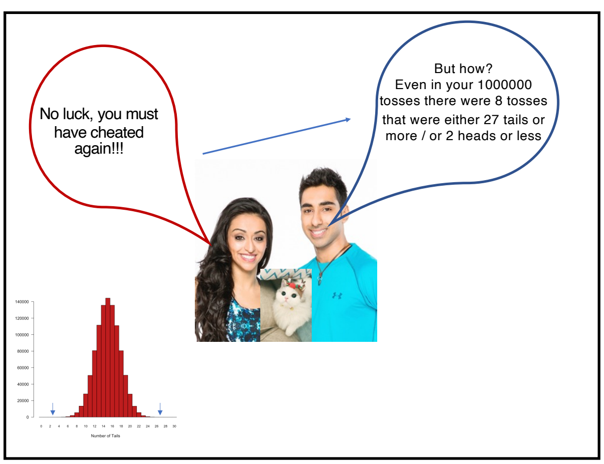
12


Nice, I got 27 tails!!! Coin was yours…and Mimy is mine!

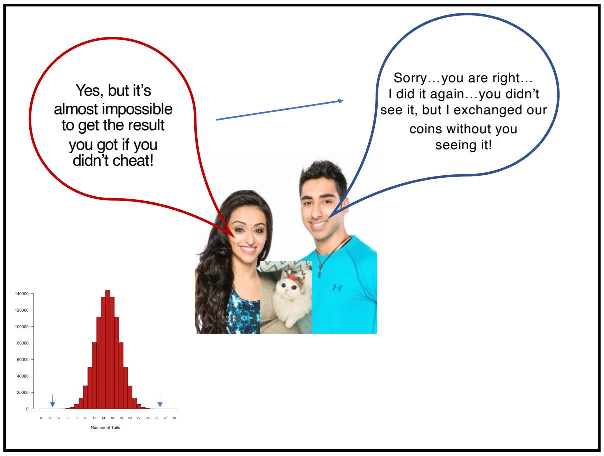
13


Hummm……
The probability of getting 27 values or more to be tails or heads is P=0.00000003 according to my fair coin!
I'm just that lucky! I guess!

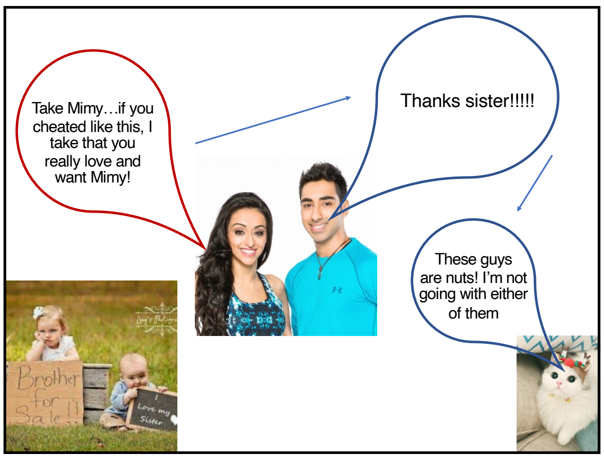
14


No luck, you must have cheated again!!!
But how? Even in your 1000000 tosses there were 8 tosses that were either 27 tails or more / or 2 heads or less

15



16



17

Using a criminal trial analogy to enhance understanding of statistical hypothesis testing: Pedagogical strategies for lasting learning.

In the US, a criminal trial makes two contradictory (mutually exclusive) claims about the defendant, either "not guilty" or "guilty". Despite these claims, the following happens:

1. The defendant is truly either "innocent" or "guilty." [these are assumptions since we never know the truth in most cases].

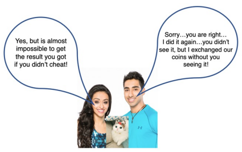
2. The defendant is then presumed "innocent until proven guilty."

3. The defendant is found guilty only if there is *strong evidence* that the defendant is guilty. The phrase "beyond a reasonable doubt" is often used as a guideline for determining a cut off for when enough evidence exists to find the defendant guilty.

4. The defendant is found to be either "not guilty" or "guilty" in the ultimate verdict.

Adapted from: https://moderndive.com/9-hypothesis-testing.html#ht-interpretation

18



The lower the p-value, the more surprising the evidence, making the null hypothesis seem increasingly implausible.

[i.e., that Jinder is not guilty].

And what did Sukhy do when she felt ridiculous about her null hypothesis?

She rejected it and chose the alternative hypothesis instead [i.e., Jinder is guilty].

Adapted from:https://towardsdatascience.com/p-values-explained-by-data-scientist-f40a746cfc8

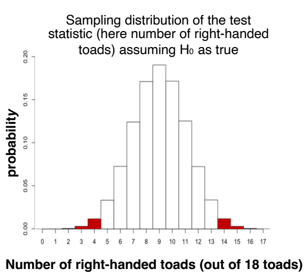
19

Let's take a break - 1 minute

20

Back to toads

We assume the null hypothesis (H_0) to be true when building the sampling distribution under H_0. Consequently, all values in the sampling distribution, including that of your sample, are considered possible. Therefore, even when we reject H_0 based on an alpha threshold, there is a chance of error since values below alpha can still occur when H_0 is true.



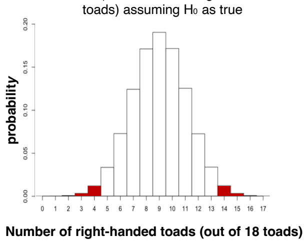
Pr[14 or more right-handed toads] = Pr[14] + P[15] + P[16] + P[17] + P[18] = 0.0155 x 2 (symmetric distribution) = 0.031

21

Back to toads

We assume the null hypothesis (H_0) to be true when building the sampling distribution under H_0. Consequently, all values in the sampling distribution, including that of your sample, are considered possible. Therefore, even when we reject H_0 based on an alpha threshold, there is a chance of error since values below alpha can still occur when H_0 is true.

Sampling distribution of the test statistic (here number of right-handed toads) assuming H_0 as true



Any test statistic value in the null distribution that is as extreme or more extreme (either larger or smaller) provides evidence against the null hypothesis.

The P-value helps determine whether the evidence supports or contradicts the statistical null hypothesis.

Therefore, the probability represented by the P-value corresponds to the likelihood of obtaining a test statistic as extreme as, or more extreme than, the observed value under the null hypothesis.

Pr[14 or more right-handed toads] = Pr[14] + P[15] + P[16] + P[17] + P[18] = 0.0155 x 2 (symmetric distribution) = 0.031

22

Statistical errors - two types of errors in statistical testing (using the criminal trial analogy)

	Reality (unknown)	
Conclusion based on sample (evidence)	H_0 true (innocent)	H_0 false (guilty)
Reject H_0 ("guilty")	Type I error	Correct
Do not reject H_0 ("not guilty")	Correct	Type II error

Type I error = FALSE POSITIVE
(its probability represent the probability of rejecting H_0 when is false, i.e., alpha).

Type II error = FALSE NEGATIVE
(its probability represents the probability of rejecting H_A when is true)

23

In statistical hypothesis testing, we assume the null hypothesis (H_0) to be true when constructing the sampling distribution.

This means that all values within the sampling distribution, including the observed sample value, are possible outcomes under the null hypothesis.

If your sample value differs significantly from the null distribution, you have reason to conclude that it is improbable under H_0 and therefore reject it.

However, keep in mind that whether you reject or fail to reject H_0, there is always a possibility of being wrong.

24

In statistical hypothesis testing, we assume the null hypothesis (H_0) to be true when constructing the sampling distribution.

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If your sample value differs significantly from the null distribution, you have reason to conclude that it is improbable under H_0 and therefore reject it.

However, keep in mind that whether you reject or fail to reject H_0, there is always a possibility of being wrong.

The protection against incorrectly rejecting a true null hypothesis (Type I error) is determined by the alpha level (significance level).

To reduce the likelihood of not rejecting a false null hypothesis (Type II error), increasing the sample size is often effective, as larger samples typically lead to smaller p-values and lower Type II error rates.

25

Critical definitions

A Type I error occurs when a true null hypothesis is incorrectly rejected (i.e., rejecting the null hypothesis when it should not be rejected). Its probability is the significance level (α), which is determined by us and remains unaffected by the sample size (n).

Type II error is failing to reject a false null hypothesis (i.e., do not reject the null hypothesis when you should not have). Its probability is β and is more complex to estimate (advanced stats). This probability decreases as sample size increases.

The power of a test ($1 - \beta$) is the probability of correctly rejecting the null hypothesis when it is truly false. This probability increases as the sample size grows.

26

Hypothesis testing involving a continuous variable; the toad problem involved a categorial variable

Normal human body temperature, as kids are taught in North America, is 98.6ºF (37ºC). But how well is this supported by data?

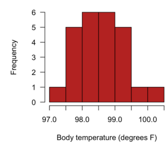
Let's understand this problem under a statistical hypothesis testing framework

27

Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data? Researchers obtained body-temperature measurements on randomly chosen healthy people (Schoemaker 1996). The data for the 25 people are as follows:

98.4	98.6	97.8	98.8	97.9
99.0	98.2	98.8	98.8	99.0
98.0	99.2	99.5	99.4	98.4
99.1	98.4	97.6	97.4	97.5
97.5	98.8	98.6	100.0	98.4

The data looks relatively symmetric so for now we have a good indication that these data are "normally" distributed. We'll see later in the course how to test this assumption in a more rigorous way.



28

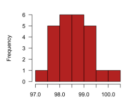
Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data?

Let's understand this problem under a statistical hypothesis testing framework

H_0 (null hypothesis): the mean human body temperature is 98.6ºF.

H_A (alternative hypothesis): the true population is different from 98.6ºF.

The Probability (or P-value), or estimated probability, is the probability of finding the observed, or more *extreme*, assuming that the null hypothesis (H_0) related to a study question (μ = 98.6ºF) is true.



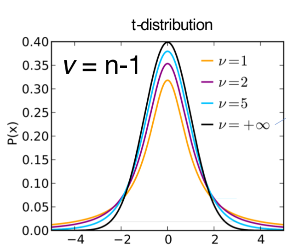
$Y = 98.524$

H_0 can be stated as "any observed difference between the sample mean (98.524ºF) and the theoretical population value (98.6ºF) is due to chance alone.

29

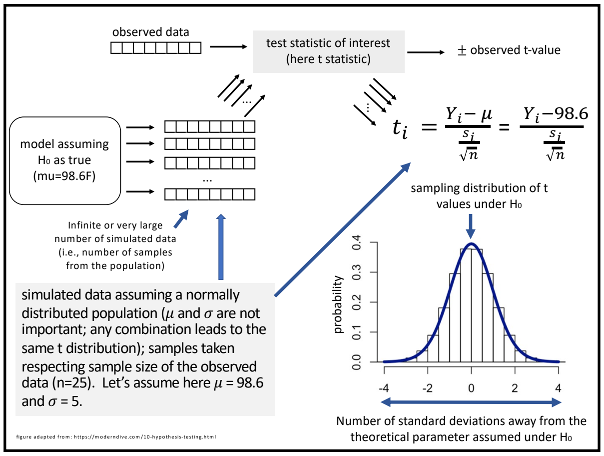
To tackle the human temperature problem, we will use the t distribution (quick review here)

For small sample sizes (n < 60), the sampling distribution of sample means from a normally distributed population are a bit far from normally distributed – the distribution is wider and depends on the sample size (degrees of freedom) – it's called the t-distribution.



Normally distributed

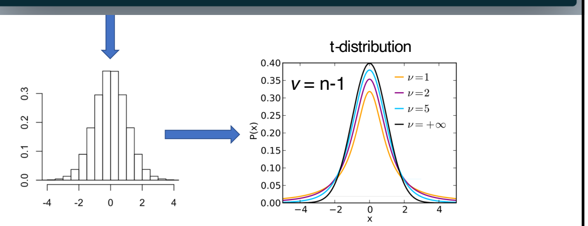
30



31

simulated data assuming a normally distributed population (μ and σ are not important; any combination leads to the same t distribution); samples taken respecting sample size of the observed data (n=25).

```
samples.pop.1 <- replicate(1000000,rnorm(n=25,mean=98.6,sd=5))
sampleMeans.Pop1 <- apply(samples.pop.1,MARGIN=2,FUN=mean)
sampleSDs.Pop1 <- apply(samples.pop.1,MARGIN=2,FUN=sd)
standardized.SampDist.Pop1 <- (sampleMeans.Pop1-98.6)/(sampleSDs.Pop1/sqrt(25))
hist(standardized.SampDist.Pop1)
```



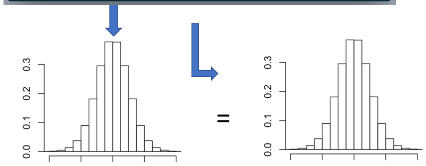
32

Two populations, two different means, two different standard deviations, but samples have the same size n

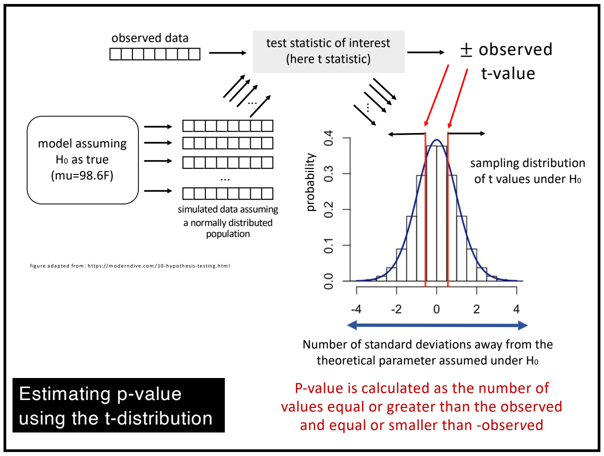
```
samples.pop.1 <- replicate(1000000,rnorm(n=25,mean=98.6,sd=5))
sampleMeans.Pop1 <- apply(samples.pop.1,MARGIN=2,FUN=mean)
sampleSDs.Pop1 <- apply(samples.pop.1,MARGIN=2,FUN=sd)
standardized.SampDist.Pop1 <- (sampleMeans.Pop1-98.6)/(sampleSDs.Pop1/sqrt(25))
hist(standardized.SampDist.Pop1)
```

```
samples.pop.2 <- replicate(1000000,rnorm(n=25,mean=13,sd=15))
sampleMeans.Pop2 <- apply(samples.pop.2,MARGIN=2,FUN=mean)
sampleSDs.Pop2 <- apply(samples.pop.2,MARGIN=2,FUN=sd)
standardized.SampDist.Pop2 <- (sampleMeans.Pop2-13)/(sampleSDs.Pop2/sqrt(25))
hist(standardized.SampDist.Pop2)
```

Because of the standardization process, the standardized t-distribution has always means 0 and the standard deviation only depends on the sample size.



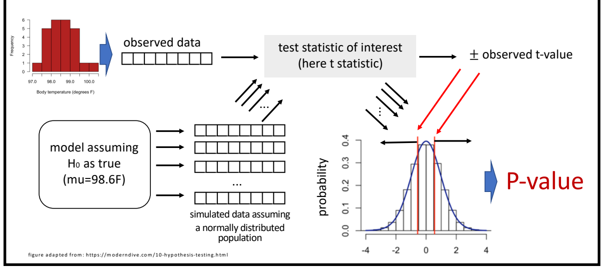
33



34

The data looks relatively symmetric so for now we have a good indication that these data are normally distributed. We'll see later in the course how to test this assumption in a more rigorous way.

We assume a normally distributed population to generate the t-distribution, either through simulation or infinite sampling calculus. However, if the original population is not normal, the standardized sampling distribution of the means may also deviate from normality, and the standard error may become biased (as previously discussed).



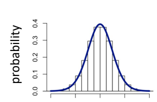
35

The use of probability distribution functions for continuous variables

Unlike discrete variables (e.g., handedness in toads), the t-distribution is primarily used for continuous variables (e.g., temperature) and is described using a probability density function (pdf). Since the probability of a specific value occurring in a pdf is zero, we instead calculate the probability of a value falling within a specified range between two points on the pdf.

Wikipedia has a great intuitive explanation about a pdf:

"Suppose a species of bacteria typically lives 4 to 6 hours. What is the probability that a bacterium lives *exactly* 5 hours? The answer is 0%. A lot of bacteria live for *approximately* 5 hours, but there is no chance that any given bacterium dies at *exactly* 5.0000000000... hours. Instead, we might ask: What is the probability that the bacterium dies between 5 hours and 5.01 hours? Let's say the answer is 0.02 (i.e., 2%)." The same applies for human temperatures and any other continuous variable (impossible to precise its value).



36

Let's take a break - 1 minute

37

One sample t-test

Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data? Researchers obtained body-temperature measurements on randomly chosen healthy people (Schoemaker 1996).
The data for the 25 people are as follows:

98.4	98.6	97.8	98.8	97.9
99.0	98.2	98.8	98.8	99.0
98.0	99.2	99.5	99.4	98.4
99.1	98.4	97.6	97.4	97.5
97.5	98.8	98.6	100.0	98.4

$Y = 98.524$ $s = 0.678$ $\mathrm{SE}_Y \frac{0.678}{\sqrt{25}} = 0.136$

38

Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data?

Let's "transform" this question into a probabilistic statement:

$$t_i = \frac{Y_i - 98.6}{\frac{s_i}{\sqrt{n}}}$$

What is the probability of obtaining a sampling *mean as extreme or more extreme* than 98.524ºF given that the theoretical population mean (assumed under H_0) is 98.6ºF?

$$t = \frac{98.524 - 98.6}{0.136} = -0.56$$

The sample mean is -0.56 standard deviations away from the mean of the theoretical population (assumed under H_0)!

39

H0 (null hypothesis): the mean human body temperature is 98.6ºF.

HA (alternative hypothesis): the true population is different from 98.6ºF.

Should we reject or not reject the H0?

$$t = \frac{98.524 - 98.6}{0.136} = -0.56$$

The sample mean is -0.56 standard deviations away from the mean of the theoretical population (assumed under H0)!

The answer lies in the probability of finding a sample value smaller or equal to -0.56 in the sampling distribution of the theoretical population assumed under H0 (μ = 0)?

40

We started with: *Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data?*

Then "translated" the above question into: What is the probability of obtaining a sampling *mean as extreme or more extreme* (i.e., *smaller*) than 98.524ºF given that the population mean is 98.6ºF?

$$t = \frac{98.524 - 98.6}{0.136} = -0.56$$

The sample mean is -0.56 standard deviations away from the mean of the theoretical population (assumed under H0)!

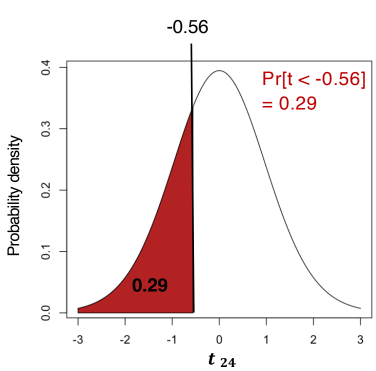
In probabilistic terms, the question then becomes: What is the probability of finding a sample t value equal or smaller than -0.56 in the sampling distribution of the theoretical population (i.e., the t-distribution; where μ = 0)?

```
> 1 - pt(q=abs(-0.56),df=24)
[1] 0.2903347
```

Pr[t < -0.56] = 0.29

41

What is the probability of obtaining a sample t-value less than or equal to -0.56 from the sampling distribution of the theoretical population (i.e., the t-distribution with μ = 0)?



42

We started with: Normal human body temperature, as kids are taught in North America, is 98.6ºF. But how well is this supported by data?

Then "translated" the above question into: What is the probability of obtaining a sampling mean as extreme or more extreme (i.e., smaller) than 98.524ºF given that the population mean is 98.6ºF?

Need to consider here:

1) In principle, our goal is not to determine whether the sample mean is smaller or larger than the theoretical population mean under the null hypothesis (98.6°F).

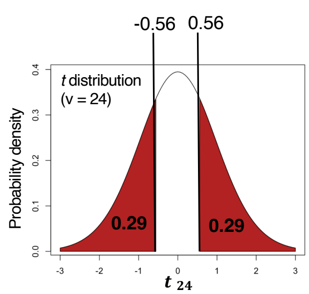
Instead, we aim to assess whether the sample mean is consistent—meaning it is a likely outcome among potential samples from the theoretical population (98.6°F)—or inconsistent, indicating it is an unlikely (uncommon) outcome under the null hypothesis. If the sample mean is inconsistent, it suggests that it does not align with the assumption that the theoretical population mean is true.

SO, WE NEED TO CONSIDER BOTH SIDES OF THE t DISTRIBUTION.

43

What is the probability of obtaining a sample t-value less than or equal to -0.56 or greater than or equal to +0.56 in the sampling distribution of the theoretical population (i.e., the t-distribution with μ = 0)?

P-value=0.58

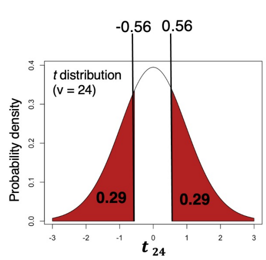


Pr[t < -0.56] + Pr[t > 0.56] =
2 Pr[t > abs(0.56)] = **0.58**
(*t* is symmetric around μ)

44

This probability 0.58 is quite large for an alpha = 0.05 (significance level). As such, we lack evidence to suggest that the observed mean does not belong to a population that has a mean (μ) = 98.6ºF.

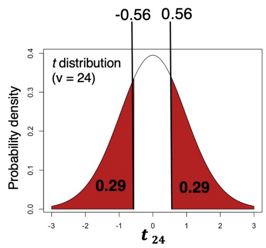
In other words, we DO NOT reject the H_0 (null hypothesis) that the mean human body temperature is 98.6ºF.



45

As such, the p-value is the evidence against the null hypothesis. The p-value is relatively large (0.58), and, as such, the evidence against H_0 is weak.

By not rejecting H_0, we cannot state that the true population value is 98.6ºF; all we can state is that we have no evidence to state that it is not (i.e., to the contrary)!



46

The Procedure for a One-Sample Mean Test

1. Establish the theoretical population mean under H0H0: This is the parameter used to standardize the sample mean and generate the tt-value (also referred to as the t-score, t-statistic, or t-deviate).

2. Take a sample from the population of interest: Ensure (or assume) that the sample follows a normal distribution.

3. Standardize the sample mean relative to the population mean established in step 1: Use the t-standardization to calculate the t-score.

4. Determine the probability of obtaining the observed tt-score or a more extreme value: This involves finding the probability in the t-distribution for values as extreme or more extreme (both smaller and larger) than the observed score. Recall that the t-distribution represents the standardized sampling distribution for the population of interest (from step 1).

5. Make a decision based on the calculated probability and the significance level (α): If the probability is smaller than the significance level, reject the null hypothesis; otherwise, do not reject it.

47

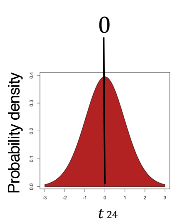
Let's think from another angle:
if the sample had a mean of 98.6ºF, then t = 0

$$t = \frac{98.6 - 98.6}{0.136} = 0$$

Pr[t < 0] + Pr[t > 0] = 1

What is the probability of obtaining a sampling mean as extreme or more extreme than 98.6ºF given that the population mean is 98.6ºF? 1.00 (100%).

Even here, by not rejecting H_0, we cannot state that the true population value is 98.6ºF; all we can say is that we have no evidence to state that it does not!



48

SUMMARY

We started with: Is the normal human body temperature of 98.6ºF, as kids are taught in North America, supported by data?

Then "translated" the above question into: What is the probability of obtaining a sampling mean as extreme or more than a sample mean of 98.524ºF given that the population mean is 98.6ºF?

1) In principle, we were not interested in knowing if the sample mean we obtained would be smaller or greater than the true population mean.

2) As such, all we are interested is to state whether we have evidence to say that the sample mean we obtained is ***consistent*** with H_0 or ***inconsistent*** with H_0.

3) If **consistent** (large P-value), then we can state that we have no evidence to state that the human temperature is different from 98.6ºF.

4) If **inconsistent** (small P-value), then we would have stated that we have evidence that the *Normal human body temperature is not 98.6ºF.*

49