

Gaining further insights into data and biological problems (experimental or observational)

Displaying numerical data in the form of frequency distributions: table and histograms & other visual aids to understand the characteristics of data.

1



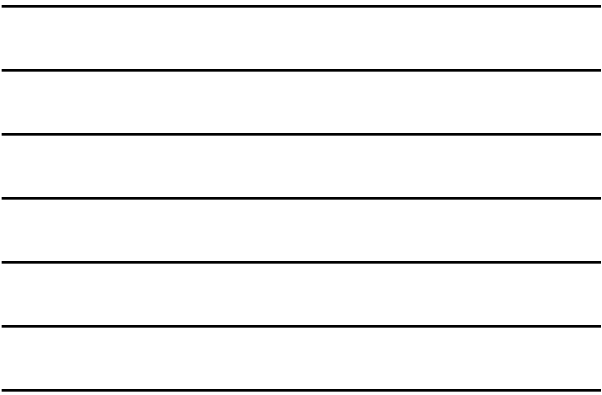
Some raw data: Abundance of birds across species

Table 2.2-2 Data on the abundance of each species of bird encountered during four surveys in Organ Pipe Cactus National Monument.

Species	Abundance	Species	Abundance
Greater roadrunner	1	Turkey vulture	23
Black-chinned hummingbird	1	Violet-green swallow	23
Western kingbird	1	Lesser nighthawk	25
Great-tailed grackle	1	Scott's oriole	28
Bronzed cowbird	1	Purple martin	33
Great horned owl	2	Black-throated sparrow	33
Costa's hummingbird	2	Brown-headed cowbird	59
Canyon wren	2	Black vulture	64
Canyon towhee	2	Lucy's warbler	67
Harris's hawk	3	Gilded flicker	77
Loggerhead shrike	3	Brown-crested flycatcher	128
Hooded oriole	4	Mourning dove	135
Northern mockingbird	5	Gambel's quail	148
American kestrel	7	Black-tailed grackle	152
Rock dove	7	Ash-throated flycatcher	173
Bell's vireo	10	Curve-billed thrasher	173
Common raven	12	Cactus wren	230
Northern cardinal	13	Verdin	282
House sparrow	14	House finch	297
Ladder-backed woodpecker	15	Gila woodpecker	300
Red-tailed hawk	16	White-winged dove	625
Phainopepla	18		

Whitlock & Schluter, *The Analysis of Biological Data*, 3e © 2020 W. H. Freeman and Company **Total = 3215 individuals**

2



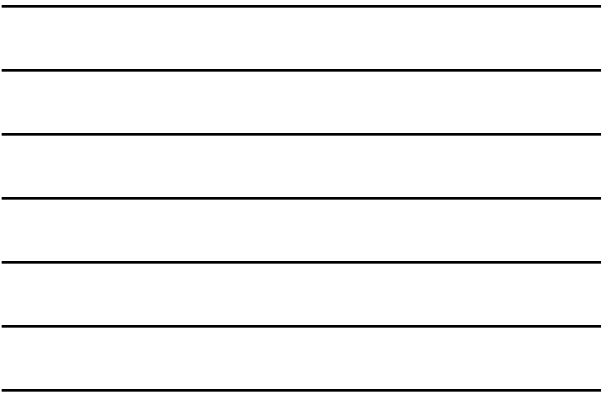
Abundance of birds across species - plot of raw data

Table 2.2-2 Data on the abundance of each species of bird encountered during four surveys in Organ Pipe Cactus National Monument.

Species	Abundance	Species	Abundance
Greater roadrunner	1	Turkey vulture	23
Black-chinned hummingbird	1	Violet-green swallow	23
Western kingbird	1	Lesser nighthawk	25
Great-tailed grackle	1	Scott's oriole	28
Bronzed cowbird	1	Purple martin	33
Great horned owl	2	Black-throated sparrow	33
Costa's hummingbird	2	Brown-headed cowbird	59
Canyon wren	2	Black vulture	64
Canyon towhee	2	Lucy's warbler	67
Harris's hawk	3	Gilded flicker	77
Loggerhead shrike	3	Brown-crested flycatcher	128
Hooded oriole	4	Mourning dove	135
Northern mockingbird	5	Gambel's quail	148
American kestrel	7	Black-tailed grackle	152
Rock dove	7	Ash-throated flycatcher	173
Bell's vireo	10	Curve-billed thrasher	173
Common raven	12	Cactus wren	230
Northern cardinal	13	Verdin	282
House sparrow	14	House finch	297
Ladder-backed woodpecker	15	Gila woodpecker	300
Red-tailed hawk	16	White-winged dove	625
Phainopepla	18		

Whitlock & Schluter, *The Analysis of Biological Data*, 3e © 2020 W. H. Freeman and Company **Stripchart "one dimensional scatter plot"**

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### Displaying numerical data in the form of frequency distributions – the tabular (table) form

Species	Abundance	Species	Abundance
Greater roadrunner	8	Yellow-crowned night heron	23
Black-chinned hummingbird	1	Violet green swallow	23
Brewer's blackbird	1	Lesser night hawk	21
Green-collared parakeet	1	Scrub wren	20
Western tanager	1	Pyrrhuloxia	19
Great horned owl	2	Black-chinned sparrow	19
Canyon towhee	2	Black-headed chickadee	18
Canyon wren	2	Black vulture	16
Harlequin duck	2	Red-winged blackbird	16
Merlin hawk	3	Golden-crowned kinglet	16
Scrub-wren sparrow	3	Scrub-wren sparrow	16
Hooded oriole	4	Whitish-tailed sparrow	15
Western tanager	5	California sparrow	148
American kestrel	7	Black-billed gnatcatcher	132
Black dove	7	Red-winged blackbird	121
Belted sparrow	10	Canyon-tailed sparrow	119
Common raven	12	Cactus wren	100
Western cardinal	13	Indigo bunting	92
House sparrow	14	House Finch	92
Ladder-backed woodpecker	15	Cinereous vulture	90
Red-tailed hawk	16	White-winged dove	615
Phainopepla	18		

Abundance	Frequency (Number of species)
0-50	28
50-100	4
100-150	3
150-200	3
200-250	1
250-300	2
300-350	1
350-400	0
400-450	0
450-500	0
500-550	0
550-600	0
600-650	1
<b>Total</b>	<b>43</b>

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### Displaying numerical data in the form of frequency distributions – from tabular to graphical form (histograms)

Abundance	Frequency (Number of species)
0-50	28
50-100	4
100-150	3
150-200	3
200-250	1
250-300	2
300-350	1
350-400	0
400-450	0
450-500	0
500-550	0
550-600	0
600-650	1
<b>Total</b>	<b>43</b>

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### The formal definitions of frequency distributions

Frequency distribution is a representation, either in a graphical or tabular format, that displays the number of observations within a given interval of a quantitative variable (continuous or discrete).

The intervals must be *mutually exclusive* (each observation can only belong to one interval) and *exhaustive* (all observations must be included),

The interval size depends on the data being analyzed and the goals of the analyst.

- Adapted from: <http://www.investopedia.com/terms/f/frequencydistribution.asp>

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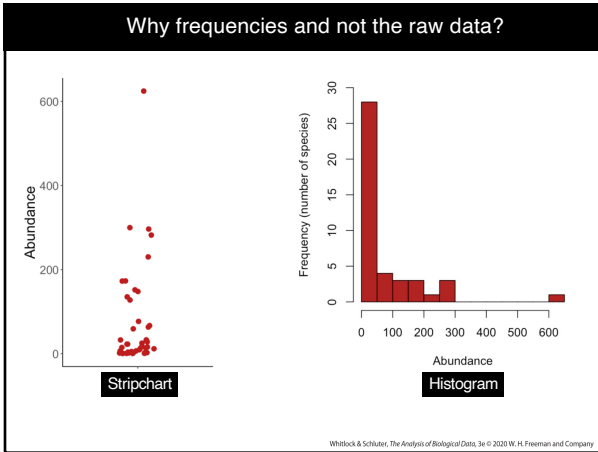
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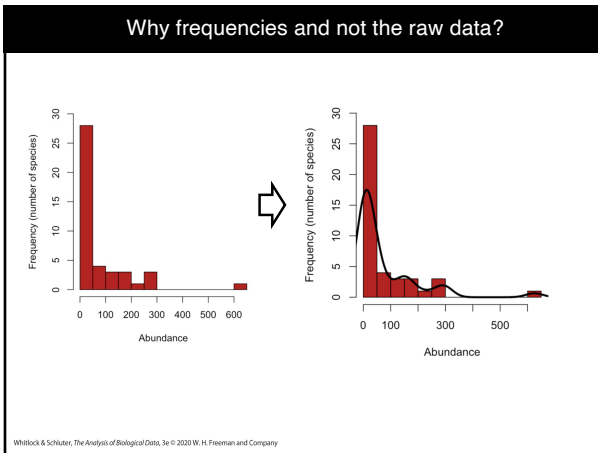
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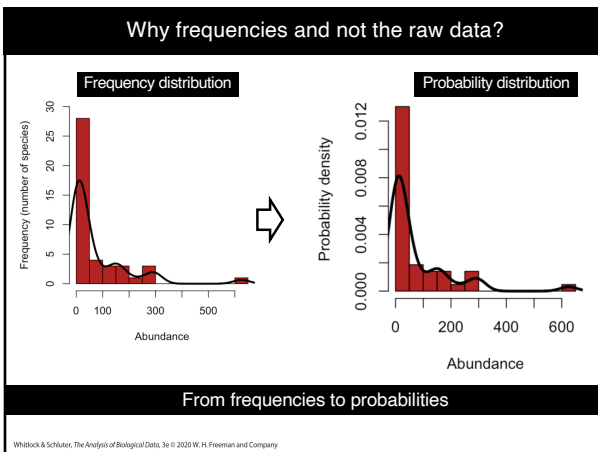
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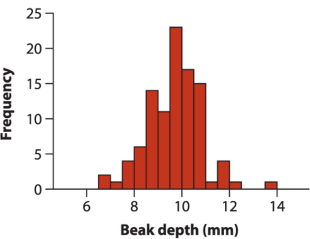
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### Why frequencies and not the raw data?



The large-beaked ground finch on the Galápagos Islands.  
*Geospiza magnirostris*



Total of 100 individual birds

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Dolph Schluter

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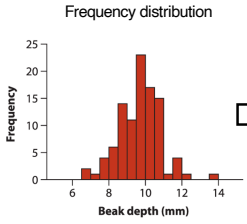
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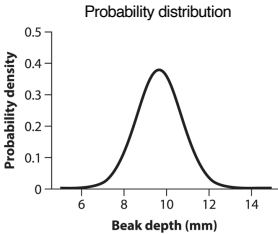
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### From frequencies to probabilities

Frequency distribution



Probability distribution



Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics (i.e., use samples to estimate population parameters; & convey uncertainty)

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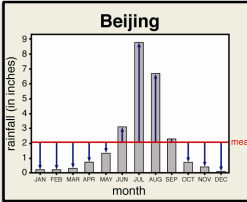
11

### Variability in bar graphs (categorical) *versus* histograms (numerical)

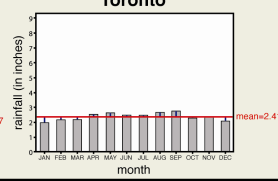
## Where does rain vary the most?

[the case of categorical variables]

**Beijing**



**Toronto**



- Source: Cooper & Shore; Journal of Statistics Education (vol. 18, #2)

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Variability in bar graphs (categorical) versus histograms (numerical)

Which class has the most variation in exam scores?  
[the case of continuous variables]

Note: scales (X and Y axis limits) are exactly the same

- Source: Cooper & Shore, Journal of Statistics Education (vol. 18, #2)

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Variability in bar graphs (categorical) versus histograms (numerical) – where do data vary the most?

mean=2.067

mean=2.417

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Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics

Some possible shapes of frequency distributions.

The **mode** is the interval corresponding to the highest peak in the frequency distribution. A distribution is said bimodal when it has two dominant peaks.

**Skew** refers to asymmetry in the shape of a frequency distribution for a numerical variable.

Mode is between 1.2 and 1.4

Intervals

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Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics

Uniform    Bell-shaped    Asymmetric (skewed)    Bimodal

Asymmetric distributions can be either left or positive skewed.

The rule based on the relationship between the mean and median is particularly effective for large datasets (greater than 30 observations).

Left (or Negative) skewed    Right (or Positive skewed)

Mean    Median    Median    Mean

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Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics

Uniform    Bell-shaped    Asymmetric (skewed)    Bimodal

Symmetric Distribution

Mean = Median

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Let's take a small break – 1 minute

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**Building a frequency distribution**

*How many intervals (classes of abundance) should be used?*

**No strict** rules need to be imposed, but rather a number that best show patterns and exceptions in data.

Body mass of 228 female sockeye salmon sampled from Pick Creek in Alaska (Hendry et al. 1999). The same data are shown in each case, but the interval widths are different : 0.1 kg (left), 0.3 kg (middle), and 0.5 kg (right).

Remember that histograms are graphical representations of frequency distributions

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**Building a frequency distribution – How many intervals?**

"Flying" paradise tree snake (*Chrysopelea paradisi*). To better understand how lift is generated, Socha (2002) videotaped glides (from a 10-m tower) of 8 snakes. Rate of side-to-side undulation was measured in hertz (number of cycles per second). The values recorded were:

**0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0**

**No strict** rules should be used, but rather a number that best show patterns and exceptions in data. Rules exist, however, example:

The Sturges' rule: number of intervals =  $1 + \ln(n) / \ln(2)$ ,

For the snake data:  $1 + \ln(8) / \ln(2) = 4$  classes.

NOTE:  $1 + \ln(n) / \ln(2) = 1 + \log_2(n)$   
(as often expressed in some sources).

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**Building a frequency distribution – The interval size**

**0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0**

Snake data:  $1 + \ln(8) / \ln(2) = 4$  classes

Let's establish the speed intervals (let's say we decide on 4 intervals):

(max(value) - min (value)) / number of classes:

**(2.0-0.9) / 4 = 0.275**

NOTE: Intervals of frequency distributions are commonly referred to as "classes" as well

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21

**Remember**

The intervals must be **mutually exclusive** (each observation can only belong to one interval) and **exhaustive** (all observations must be included), and the interval size depends on the data being analyzed and the goals of the analyst.

- Adapted from: <http://www.investopedia.com/terms/f/frequencydistribution.asp>

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**Building intervals**

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0 - 0.9) / 4 = 0.275$

1<sup>st</sup> class: individuals with speeds between 0.900 and 1.175 ( $0.900 + 0.275$ )

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**Building intervals**

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0 - 0.9) / 4 = 0.275$

1<sup>st</sup> class: individuals with speeds between 0.900 and 1.175 ( $0.900 + 0.275$ )

2<sup>nd</sup> class: individuals with speeds between 1.175 and 1.450 ( $1.175 + 0.275$ )

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**Building intervals**

*Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0*

(max(value) - min (value)) / number of classes:

$(2.0-0.9) / 4 = \underline{0.275}$

1<sup>st</sup> class: individuals with speeds between 0.900 and 1.175 ( $0.900 + 0.275$ )

2<sup>nd</sup> class: individuals with speeds between 1.175 and 1.450 ( $1.175 + 0.275$ )

3<sup>rd</sup> class: individuals with speeds between 1.450 and 1.725 ( $1.450 + 0.275$ )

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**Building intervals**

*Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0*

(max(value) - min (value)) / number of classes:

$(2.0-0.9) / 4 = \underline{0.275}$

1<sup>st</sup> class: individuals with speeds between 0.900 and 1.175 ( $0.900 + 0.275$ )

2<sup>nd</sup> class: individuals with speeds between 1.175 and 1.450 ( $1.175 + 0.275$ )

3<sup>rd</sup> class: individuals with speeds between 1.450 and 1.725 ( $1.450 + 0.275$ )

4<sup>th</sup> class: individuals with speeds between 1.725 and 2.000 ( $1.725 + 0.275$ )

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**Counting number of observations (frequencies)**

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's use: left-closed & right-open [a,b)

Classes	Frequency
0.900 - 1.175	
1.175 - 1.450	
1.450 - 1.725	
1.725 - 2.000	

Intervals are either left-closed & right-open, e.g., 0.900 - 1.175 would contains snakes with rates between 0.9 Hz (included) and 1.175 Hz (not included) = [0.900,1.175).

OR left-open & right-closed, e.g., 0.900 - 1.175 would contains snakes with rates between 0.9 Hz (not included) and 1.175 Hz (included) = (0.900,1.175].

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	
[1.450 - 1.725)	
[1.725 - 2.000)	

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28

Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	5
[1.450 - 1.725)	
[1.725 - 2.000)	

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29

Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
0.900 - 1.175	1
1.175 - 1.450	5
1.450 - 1.725	1
1.725 - 2.000	

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0 ?

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	5
[1.450 - 1.725)	1
[1.725 - 2.000)	???

**FAILED**

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Counting number of observations (frequencies)

? 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's try left-open & right-closed (a,b]

Classes	Frequency
(0.900 - 1.175]	???
(1.175 - 1.450]	
(1.450 - 1.725]	
(1.725 - 2.000]	

**FAILED**

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Counting number of observations (frequencies)

Let's try a different number of classes (5) and interval size (0.275)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)		left-open & right-closed (a,b]	
Classes	Frequency	Classes	Frequency
[0.900 - 1.175)	1	(0.625 - 0.900]	1
[1.175 - 1.450)	5	(0.900 - 1.175]	0
[1.450 - 1.725)	1	(1.175 - 1.450]	5
[1.725 - 2.000)	0	(1.450 - 1.725]	1
[2.000 - 2.275)	1	(1.725 - 2.000]	1

It works, but the classes may not print well. They have too many decimals. We can change the number of classes to try to fix this issue (let's try 7 classes next).

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**Counting number of observations (frequencies)**

Let's try a different number of classes (7) and interval size (0.2)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's use: left-closed & right-open [a,b)

Classes	Frequency
[0.8 - 1.0)	1
[1.0 - 1.2)	0
[1.2 - 1.4)	3
[1.4 - 1.6)	2
[1.6 - 1.8)	1
[1.8 - 2.0)	0
[2.0 - 2.2)	1
<b>Total</b>	<b>= 8</b>

Note: some software may include 2.0 in this interval even though is opened. This may happen when the last values in the data fall here. (R does that)

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**From frequency distribution tables to histograms**

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.8 - 1.0)	1
[1.0 - 1.2)	0
[1.2 - 1.4)	3
[1.4 - 1.6)	2
[1.6 - 1.8)	1
[1.8 - 2.0)	0
[2.0 - 2.2)	1

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**From frequency distribution tables to histograms**

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-open & right-closed (a,b]

Classes	Frequency
(0.80 - 1.00]	1
(1.00 - 1.20]	2
(1.20 - 1.40]	3
(1.40 - 1.60]	1
(1.60 - 1.80]	0
(1.80 - 2.00]	1

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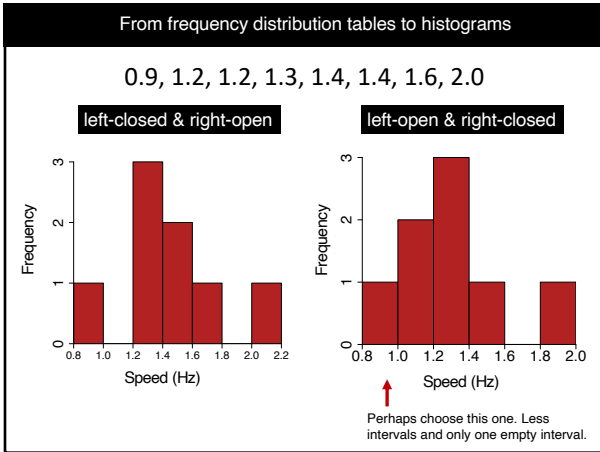
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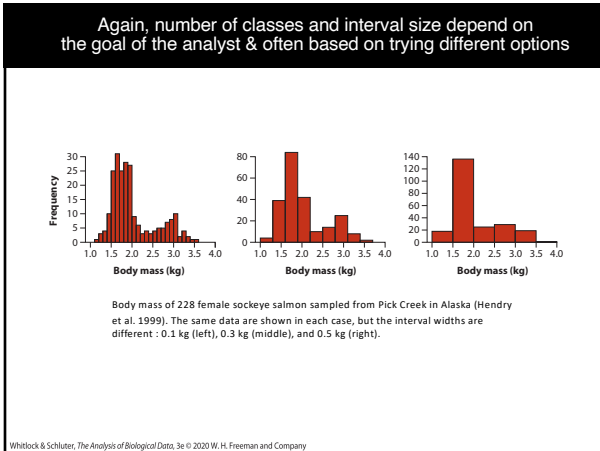
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Next lecture: describing data

Samples and populations are often made of lots of individual (observational) units and their associated information (observations, variables).

We need to be able to describe samples by summary statistics (mean, median, variance, etc) so that these summaries can serve as an estimate of the same summaries for their statistical populations.

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