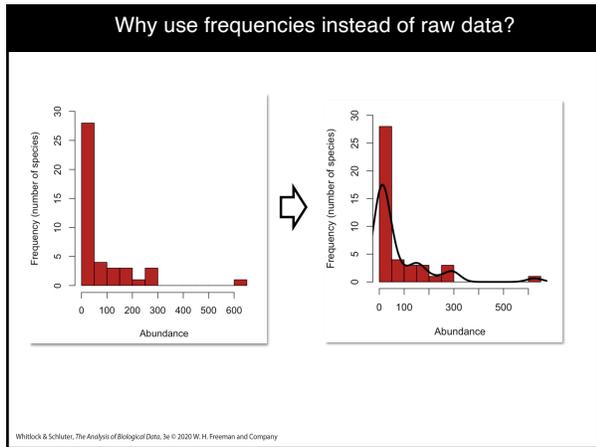
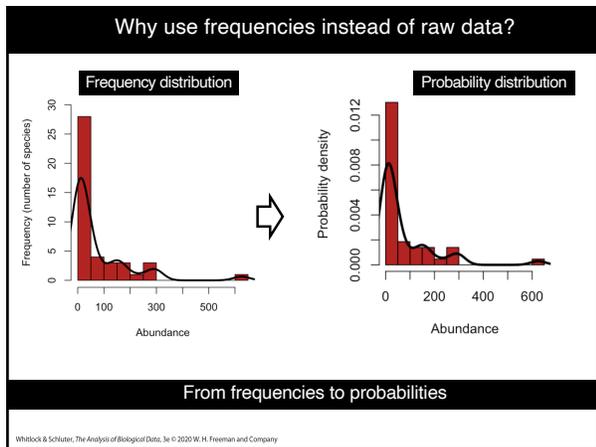


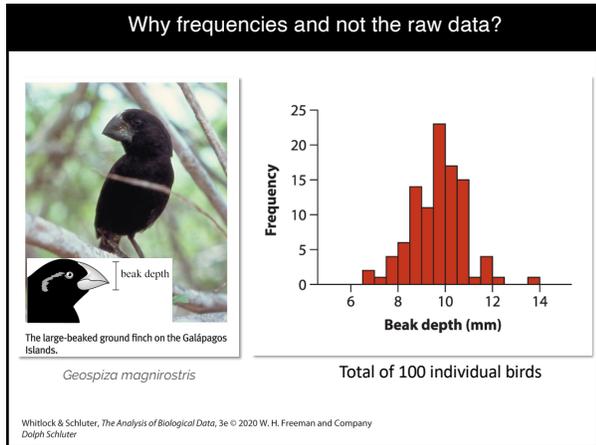
7



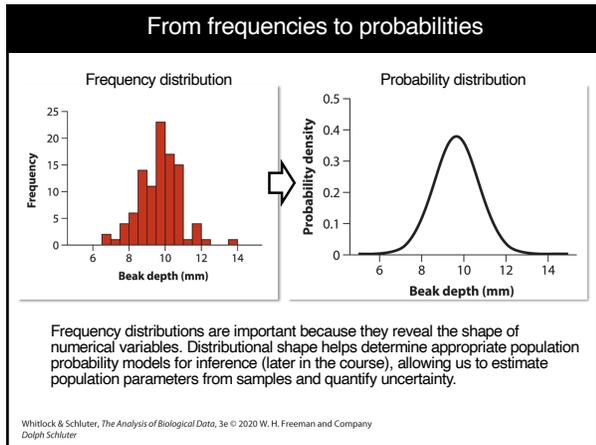
8



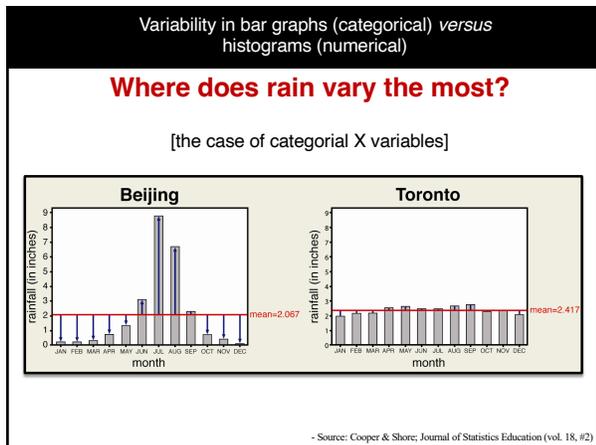
9



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Variability in bar graphs (categorical) versus histograms (numerical)

Which class has the most variation in exam scores?

[the case of continuous X variables]

Note: scales (X and Y axis limits) are the same

- Source: Cooper & Shore; Journal of Statistics Education (vol. 18, #2)

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Variability in bar graphs (categorical) versus histograms (numerical) – where do data vary the most?

Beijing mean=2.067

Toronto mean=2.417

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Frequency distributions are important because they reveal the shape of numerical variables. Distributional shape helps determine appropriate population probability models for inference (later in the course), allowing us to estimate population parameters from samples and quantify uncertainty.

Some possible shapes of frequency distributions.

The **mode** is the **interval** corresponding to the highest peak in the frequency distribution. A distribution is said bimodal when it has two dominant peaks.

Skew refers to asymmetry in the shape of a frequency distribution for a numerical variable.

Mode is between 1.2 and 1.4

Intervals

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Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics

Uniform Bell-shaped Asymmetric (skewed) Bimodal

Asymmetric distributions can be either left or positive skewed.

The rule based on the relationship between the mean and the median is particularly effective for large datasets (more than 30 observations).

Left (or Negative) skewed Right (or Positive) skewed

Mean Median Median Mean

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Frequency distributions are important because they describe shapes of numerical variables. Distributional shapes allow to determine proper population probability distributions for inferential statistics

Uniform Bell-shaped Asymmetric (skewed) Bimodal

Symetric Distribution

Mean = Median

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Let's take a small break – 1 minute

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Building a frequency distribution

How many intervals (classes of abundance) should be used?

No strict rules need to be imposed, but rather a number that best show patterns and exceptions in data.

Body mass of 228 female sockeye salmon sampled from Pick Creek in Alaska (Hendry et al. 1999). The same data are shown in each case, but the interval widths are different : 0.1 kg (left), 0.3 kg (middle), and 0.5 kg (right).

Remember that histograms are graphical representations of frequency distributions

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Building a frequency distribution – How many intervals?

"Flying" paradise tree snake (*Chrysopelea paradisi*). To better understand how lift is generated, Socha (2002) videotaped glides (from a 10-m tower) of 8 snakes. Rate of side-to-side undulation was measured in hertz (number of cycles per second). The values recorded were:

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

No strict rules should be used, but rather a number that best show patterns and exceptions in data. Rules exist, however, example:

The Sturges' rule: number of intervals = $1 + \ln(n) / \ln(2)$,

For the snake data: $1 + \ln(8) / \ln(2) = 4$ classes.

NOTE: $1 + \ln(n) / \ln(2) = 1 + \log_2(n)$
(as often expressed in some sources).

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Building a frequency distribution – The interval size

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Snake data: $1 + \ln(8) / \ln(2) = 4$ classes

Let's establish the speed intervals (let's say we decide on 4 intervals):

(max(value) - min (value)) / number of classes:

(2.0-0.9) / 4 = 0.275

NOTE: Intervals of frequency distributions are commonly referred to as "classes" as well

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Important

The intervals must be mutually exclusive, meaning that each observation can belong to only one interval, and exhaustive, meaning that all observations must be included.

The choice of interval size depends on the data being analyzed and on the goals of the analyst.

Adapted from: <http://www.investopedia.com/terms/f/frequencydistribution.asp>

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Building intervals

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0 - 0.9) / 4 = \underline{0.275}$

1st class: individuals with speeds between 0.900 and 1.175 ($0.900 + 0.275$)

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Building intervals

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0 - 0.9) / 4 = \underline{0.275}$

1st class: individuals with speeds between 0.900 and 1.175 ($0.900 + 0.275$)

2nd class: individuals with speeds between 1.175 and 1.450 ($1.175 + 0.275$)

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Building intervals

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0-0.9) / 4 = \underline{0.275}$

1st class: individuals with speeds between 0.900 and 1.175 (0.900 + 0.275)

2nd class: individuals with speeds between 1.175 and 1.450 (1.175 + 0.275)

3rd class: individuals with speeds between 1.450 and 1.725 (1.450 + 0.275)

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Building intervals

Let's establish the speed intervals: 0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

(max(value) - min (value)) / number of classes:

$(2.0-0.9) / 4 = \underline{0.275}$

1st class: individuals with speeds between 0.900 and 1.175 (0.900 + 0.275)

2nd class: individuals with speeds between 1.175 and 1.450 (1.175 + 0.275)

3rd class: individuals with speeds between 1.450 and 1.725 (1.450 + 0.275)

4th class: individuals with speeds between 1.725 and 2.000 (1.725 + 0.275)

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's use: left-closed & right-open [a,b)

Classes	Frequency
0.900 - 1.175	
1.175 - 1.450	
1.450 - 1.725	
1.725 - 2.000	

Intervals are either left-closed & right-open, e.g., 0.900 - 1.175 would contains snakes with rates between 0.9 Hz (included) and 1.175 Hz (not included) = [0.900,1.175).

OR left-open & right-closed, e.g., 0.900 - 1.175 would contains snakes with rates between 0.9 Hz (not included) and 1.175 Hz (included) = (0.900,1.175].

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	
[1.450 - 1.725)	
[1.725 - 2.000)	

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	5
[1.450 - 1.725)	
[1.725 - 2.000)	

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
0.900 - 1.175	1
1.175 - 1.450	5
1.450 - 1.725	1
1.725 - 2.000	

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Counting number of observations (frequencies)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, **2.0** ?

left-closed & right-open [a,b)

Classes	Frequency
[0.900 - 1.175)	1
[1.175 - 1.450)	5
[1.450 - 1.725)	1
[1.725 - 2.000)	???

FAILED

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Counting number of observations (frequencies)

? **0.9**, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's try left-open & right-closed (a,b]

Classes	Frequency
(0.900 - 1.175]	???
(1.175 - 1.450]	
(1.450 - 1.725]	
(1.725 - 2.000]	

FAILED

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Counting number of observations (frequencies)

Let's try a different number of classes (5) and interval size (0.275)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)		left-open & right-closed (a,b]	
Classes	Frequency	Classes	Frequency
[0.900 - 1.175)	1	(0.625 - 0.900]	1
[1.175 - 1.450)	5	(0.900 - 1.175]	0
[1.450 - 1.725)	1	(1.175 - 1.450]	5
[1.725 - 2.000)	0	(1.450 - 1.725]	1
[2.000 - 2.275)	1	(1.725 - 2.000]	1

It works, but the class intervals may not display well because they include too many decimal places. We can adjust the number of classes to address this issue—let's try using seven classes next.

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Counting number of observations (frequencies)

Let's try a different number of classes (7) and interval size (0.2)

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

Let's use: left-closed & right-open [a,b)

Classes	Frequency
[0.8 - 1.0)	1
[1.0 - 1.2)	0
[1.2 - 1.4)	3
[1.4 - 1.6)	2
[1.6 - 1.8)	1
[1.8 - 2.0)	0
[2.0 - 2.2)	1
Total	= 8

Note: some software may include 2.0 in this interval even though is opened. This may happen when the last values in the data fall here. (R does that)

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From frequency distribution tables to histograms

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

left-closed & right-open [a,b)

Classes	Frequency
[0.8 - 1.0)	1
[1.0 - 1.2)	0
[1.2 - 1.4)	3
[1.4 - 1.6)	2
[1.6 - 1.8)	1
[1.8 - 2.0)	0
[2.0 - 2.2)	1

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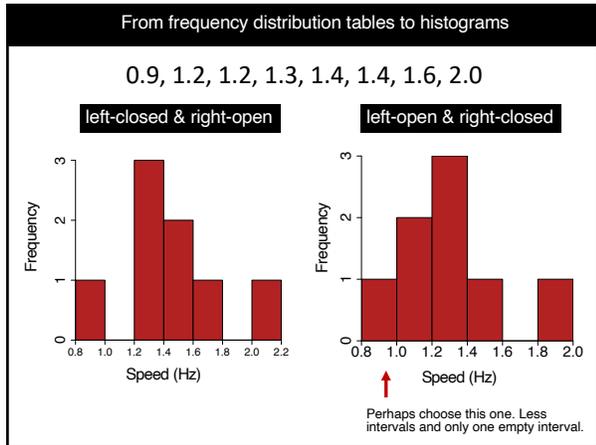
From frequency distribution tables to histograms

0.9, 1.2, 1.2, 1.3, 1.4, 1.4, 1.6, 2.0

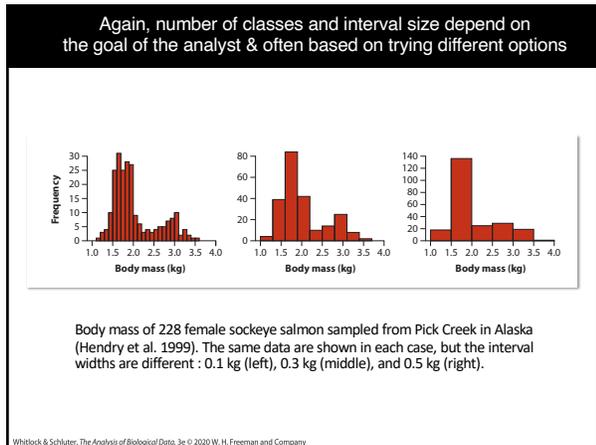
left-open & right-closed (a,b]

Classes	Frequency
(0.80 - 1.00]	1
(1.00 - 1.20]	2
(1.20 - 1.40]	3
(1.40 - 1.60]	1
(1.60 - 1.80]	0
(1.80 - 2.00]	1

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Next lecture: describing data

Samples and populations are often composed of many individual observational units, each associated with one or more measured variables.

To describe samples efficiently, we rely on summary statistics (e.g., mean, median, variance), which serve as estimates of the corresponding quantities in the underlying population.

The figure shows a 3D visualization of data points represented as blue cubes arranged in a grid that recedes into the distance, creating a perspective effect.

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